6 Graphing and Solving Equations

CHAPTER 6 Graphing and Solving Equations

In earlier chapters, you used histograms and box-and-whisker plots to answer questions such as "*Are this year's frogs better jumpers than last year's?"* In this chapter, you will develop a way to see if there is a relationship between two different measures for a set of objects. For example, to answer the question, "*If a frog has longer legs, can it jump farther?"* you will use a new tool, called a scatterplot.

In this chapter, you will also extend what you know about comparing expressions to include those cases when expressions are equal. You will build equations on equation mats with algebra tiles as well as write equations with variables and solve them without tiles. This skill will provide a new way to solve word problems without completing a 5-D table. By the end of the chapter you will learn some efficient ways to justify the steps used to solve equations.

Guiding Questions

Think about these questions throughout this chapter:

What would a graph of this data look like?

Can I make a prediction?

Is there a relationship?

How can I represent it?

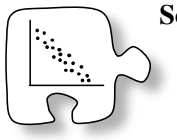
How can I solve it?

How can I check my answer?

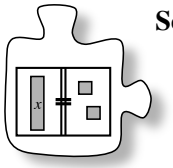
In this chapter, you will learn how to:

- Create scatterplots that show relationships among two-variable data.
- Identify correlations between sets of data and represent the relationship with a trend line.
- Solve for a variable when two expressions are equal.
- Write and solve an equation to solve a word problem.
- Recognize when an equation has no solution or infinite solutions.

Chapter Outline

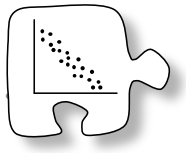


Section 6.1 In this section you will create and interpret circle graphs. You will also learn to make graphs that compare two sets of data. Then, you will use scatterplots and linear graphs to make observations and predictions about the data based on correlations.



Section 6.2 Using algebra tiles, you will explore what you can learn when expressions are equal. Solving equations will also provide you an opportunity to develop efficient simplification strategies and learn how to know that your solution is correct. You will also consider special cases, such as when an equation has no solution.

6.1.1 How can I represent the data?



Circle Graphs

Data can be found everywhere in the world. When scientists conduct experiments, they collect data. Advertising agencies collect data to learn which products consumers prefer. In Chapter 4 you developed histograms and box-and-whisker plots to represent measures (such as lengths of frog jumps). However, how can you represent data that is non-numerical or that cannot be represented on a number line? Today you will look at a data display that is used for data that comes in categories or groups. As you work, keep these questions in mind:

What portion is represented?

Should I use a fraction or a percent?

Am I measuring in percents or degrees?

6-1. HUMAN CIRCLE GRAPH

Get a shape card from your teacher. Look at your shape and decide if your shape is a parallelogram, another form of quadrilateral, a triangle, or some other shape. Follow your teacher's directions to create a linear model and circle graph. Then answer the questions below.

a. Your class built a circle graph with your bodies. How can this model be drawn on paper? Work with your team to sketch a picture of your class circle graph showing the portion of your class that held parallelograms, other quadrilaterals, triangles, and other polygons. Be sure to label each section with the category of shape it represents and with an estimated percentage or angle measure.

b. Approximately what portion of the class held triangles? Write your answer as a percent. Then estimate the measure of the central angle on the graph for that portion. A **central angle** is an angle with its vertex at the center of a circle. Its sides are formed by two radii and its measure is a portion of 360°.

c. Was there a section of the circle that had a central angle that was approximately 90°? If so, what type of figure is represented in that section?

6-2. Nate and Nick are interested in buying a car. According to an ad in the paper, they found that there were 12 cars, 9 pickup trucks, 6 SUVs, and 3 minivans for sale in their price range.

a. How many vehicles were in Nate and Nick's price range?

b. What portion of the total does each type of vehicle represent?

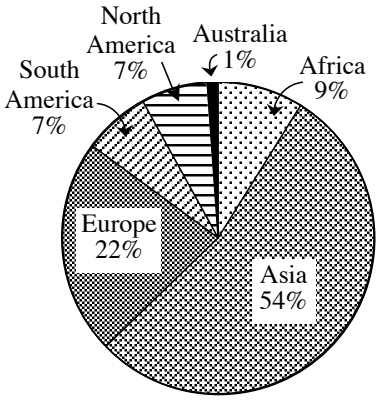
c. On the Lesson 6.1.1B Resource Page, create a circle graph of the vehicles in Nate and Nick's price range. Label each section of the graph with the type of vehicle along with the fraction or percent of the circle it represents.

d. Calculate the central angle created by each section in the circle graph.

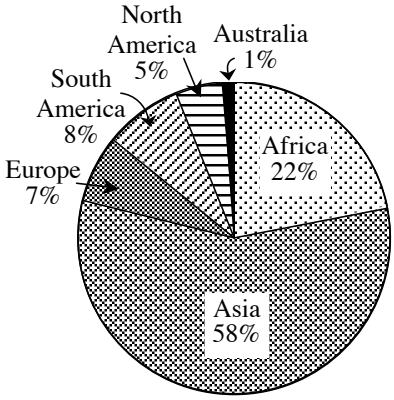
e. Is there another way to represent this data? Are a box-and-whisker plot, stem-and-leaf plot, histogram, or bar graph an appropriate way to display this data? Why or why not?

6-3. Circle graphs can be used to compare data at different points in time. Analyze the two circle graphs below.

Population of the Continents in 1950



Predicted Population of the Continents in 2050



a. According to the circle graphs, which continent had the largest population in 1950? Which has the largest predicted population in 2050? Do they represent the same percent of the world's population in both graphs?

b. Which continent is predicted to have its percentage of world population increase the most between 1950 and 2050? By how much is its percentage expected to increase?

Problem continues on next page. →

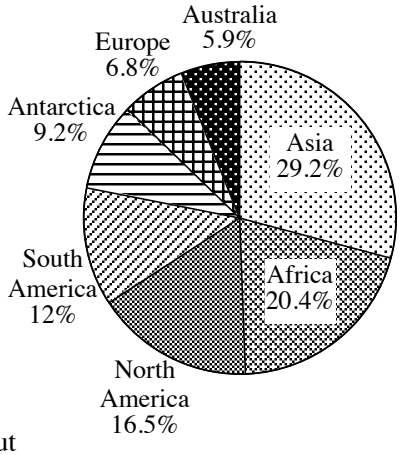
6-3. *Problem continued from previous page.*

c. Which continent is expected to have its portion of the total population shrink the most between 1950 and 2050? By how much will its percentage of world population change?

d. Is it reasonable to say that continents with a small percentage of the world population in 1950 will have small percentage of the populatation in 2050? What evidence from the graphs can you provide to **justify** your answer?

6-4. The world's landmasses are divided into seven continents. The largest continent in terms of landmass is Asia, representing almost 30% of the Earth's land. In contrast, the smallest continent is Australia at about 6% of the Earth's land. Use the circle graph at right to help you make the following comparisons.

Landmass of Continents



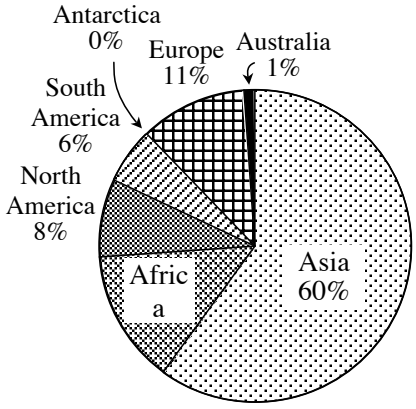
a. Which continents are approximately the same size?

b. Which continent is about one-half the size of Asia?

c. Which continents together make up about half of the world's land mass?

6-5. The population of the world's people is not evenly divided over the Earth's surface. In 2009, only 0.0002% of the people in the world lived in Antarctica, while 60% of people lived in Asia.

Population of Continents 2000

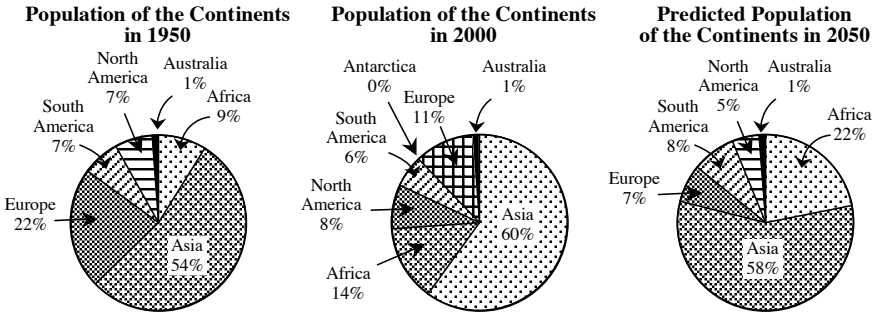


a. Where is the portion representing Antarctica's population? Explain.

b. What similarities and differences do you notice about the landmass and population circle graphs in problem 6-4 and this problem?

c. Is it reasonable to say that larger continents have larger populations? Why or why not?

6-6. **Additional Challenge:** Take another look at the population graphs from problems 6-3 and 6-5.



a. Note that Asia's population percentage goes from 54% to 60% to 58%. Does this mean the population of Asia is expected to shrink between 2000 and 2050? Why or why not?

b. What kind of graph could be used so that this misunderstanding would not occur?

METHODS AND MEANINGS

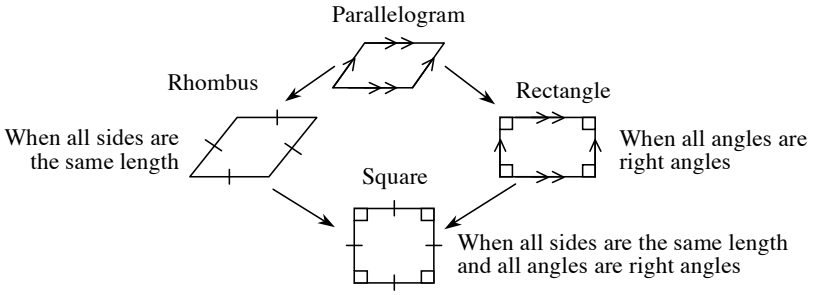
Special Quadrilaterals

MATH NOTES

Every rectangle is a parallelogram because opposite pairs of sides are parallel, but a rectangle is a special kind of parallelogram because it has four right angles. So every rectangle is a parallelogram, but not every parallelogram is a rectangle.

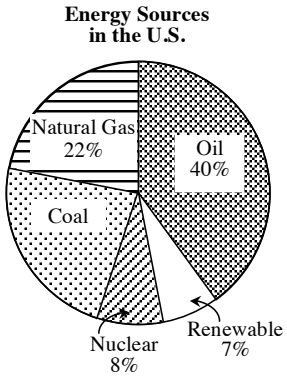
In the same way, every rhombus is a parallelogram and every square is a rhombus, a rectangle and a parallelogram.

The diagram below shows the relationships between some kinds of quadrilaterals and some of their properties. The arrows indicate that the properties of the previous figure(s) are also properties of that figure.



Review & Preview

6-7. Answer the following questions about the graph at right.



a. According to this graph, what percentage of the energy in the United States comes from coal?

b. Which two sources of energy equal about one-third of the total energy for the United States?

c. What combination of energy sources provide about half of the total energy for the United States?

6-8. If you can travel 156 miles on 4 gallons of gasoline, how far can you travel on 12 gallons? How many miles on 6 gallons? A diagram may help you with your **reasoning**. Show your work and explain your thinking.

6-9. Use the Distributive Property to simplify the following expressions.

a. $4(x+2)$ b. $-5(9+x)$ c. $7(x-3)$

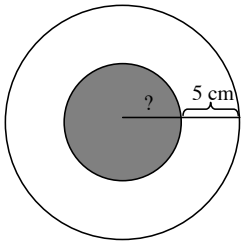
6-10. In 2009, the federal government budget was \$3.1 trillion (\$3,100,000,000,000). The government was looking to cut costs.

a. If it decided to cut 1%, how much did it cut?

b. If the government reduced the budget by 7%, how much did it cut?

c. If the government eliminated \$93 billion (\$93,000,000,000) from the budget, what percentage did it cut?

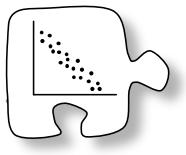
6-11. The diameter of the large circle is 18 cm.



a. What is the area of the shaded circle?

b. What steps did you do need to do to find the answer?

6.1.2 Is there a relationship?



Organizing Data in a Scatterplot

In previous chapters and Lesson 6.1.1, you have been looking at single data sets, such as world population. Often we need to compare two measurements to answer a question or to see a connection between two types of data. For example, comparing the odometer reading of a car to the price of a car can help determine if these factors are related. In this lesson you will study scatterplots, a new tool for visually presenting data, as a way to relate two sets of measurements. You will be asked to analyze the data to see if you can make predictions or come to any conclusion about the relationships that you find.

As you work with your team today, use these focus questions to help direct your discussion:

How can I organize data?

Can I use this data to make a prediction?

What does a point represent?

Is there a connection between the two factors?

6-12. HOW MUCH IS THAT CAR?

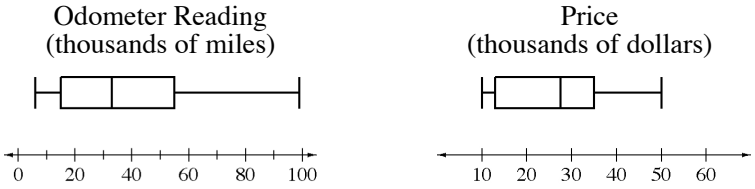
Nate and Nick were discussing cars. Nate claimed that cars with lower odometer readings were more expensive than cars with higher odometer readings. His evidence was that his car with 23,000 miles was worth more than Nick's car with 31,000 miles. To investigate Nate's claim, the boys researched several car ads and found the information in the table at right.

Nate's Data from Car Ads

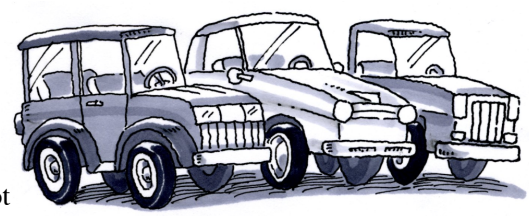
Odometer Reading (thousands of mi)	Price (thousands of $)
35	$38
55	$16
6	$50
28	$30
50	$26
31	$35
15	$28
99	$10
99	$13

Does the information in the table support Nate's claim? That is, do you believe Nate's claim that cars with a lower odometer reading cost more money?

6-13. Melissa looked at the data from problem 6-12 and said, *"I need to be able to see the data as a picture. I can't tell if there is a relationship from the lists of numbers."* She decided to use a box-and-whisker plot. Her box-and-whisker plots for odometer reading and price are shown below. Do these pictures help you decide if Nate is correct? Why or why not?



6-14. Melissa wondered if a coordinate graph could help determine if there was a relationship in Nate's data from problem 6-12.



a. Follow your teacher's directions to create a scatterplot of the data for Melissa.

- Set up a graph showing Odometer Reading on the *x*-axis and Price on the *y*-axis.
- Label equal intervals on each axis so that all of the data will fit on the graph.
- Plot the data points from problem 6-12.

b. Describe the scatterplot you just created. What do you notice about how the points are placed on the graph? Do you see any patterns?

c. Place an additional point on your graph for Nate's car that has an odometer reading of 23,000 miles. Explain your **strategy** for deciding where to put the point.

d. When a relationship exists, one way to help show a trend in the data is to place a line or curve that in general represents where the data falls. This line (sometimes called a **trend line**) does not need to touch any of the actual data points, but instead shows where the data generally falls. The line is the set of points that represents your predictions for all cars no matter what their mileage or price.

With your team, decide where a trend line could be placed that would best represent the data points. Are there any limits to when the trend line makes sense?

Problem continues on next page. →

6-14. *Problem continued from previous page.*

e. Using the trend line, can you predict the price of a car with an odometer reading of 80,000 miles? If so, explain how the trend line helps. If not, explain why it is not helpful.

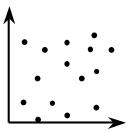
f. Based on the scatterplot, would you agree with Nate's claim that cars with a higher odometer reading cost less? Use the scatterplot to **justify** your answer.

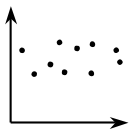
6-15. Sometimes what you know about relationships can help you predict what data will look like when it is graphed. For each situation below:

- Look at the scatterplots and use your experience to decide which statement fits each scatterplot.
- Decide if there is a relationship between the data, that is, as one quantity changes, does the other change in a predictable way?
- If there is a relationship, describe it in a sentence.
- If there is no relationship, explain why you think so.

i. 

ii. 

iii. 

iv. 

a. How fast a dog can run and the length of the dog's fur.

b. A person's age and their body temperature.

c. The child's age and the size of his or her feet.

d. Outdoor temperature and the percentage of people wearing long sleeve shirts.

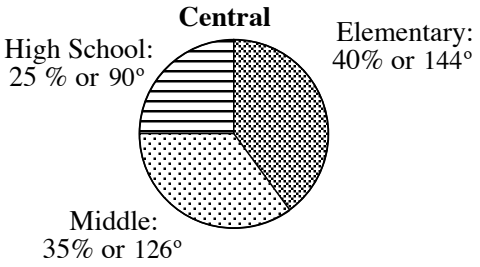
METHODS AND MEANINGS

MATH NOTES

Circle Graphs

A **circle graph** (sometimes called a **pie chart**) is like a bar graph because it deals with categorical data (such as make of car or grade in school) and not continuous data, such as age or height.

Each category of data is put into its own sector of the circle. The measure of the central angle bounding the sector is proportional to the percentage of elements of that type of the whole. For example, if Central Schools has 40% of its students in elementary school, 35% in middle school, and 25% in high school, then its circle graph would have a central angle of 144° (0.4 times 360°) for the sector showing the elementary school, 126° for the sector showing the elementary school, and 90° for the sector showing the high school.



6-16. A class surveyed how students at their school travel to school. The results of the survey are shown in the table at right. Make a circle graph showing the results of the survey using percentages.

Mode of Transportation	Number of Students
Bus	90
Ride Bike	30
Ride in Car	75
Walk	45

6-17. Simplify the following expressions.

a. $-5+2(8-12)$ b. $(-5+2)(8-12)$ c. $-5+2\cdot 8-12$

d. $\frac{1}{2}(-6)(4+10)$ e. $-\frac{2}{3}\cdot 6+15\div(-3)$ f. $(7-2)^2-5\div 5$

6-18. A triangle has a base that is two more than three times the length of its height. The area of the triangle is 400 square cm. What are the base and height of the triangle?

6-19. Alan wants to convince his parents that he is doing well in math. His test scores are 83, 90, 58, 71, 82, 91, 82, and 64. Should he use the mean, median, or mode to convince them? Explain your **reasoning** and show your work.

6-20. Find the lengths of the missing sides on the similar shapes at right.